Extending the Higgs Effective Theory

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October 29, 2009
LHC Fellows Meeting
FermiLab
NSF grant PHY-0705682

A Work Horse

The effective Lagrangian $L_{int} = C\frac{HF^a_{\mu\nu}F^{a\mu\nu}}{4v}$ has dominated Higgs production physics calculations for well known reasons.

- It is the *only* (CP even) operator at leading order in the heavy mass expansion of the top quark.
- The matching coefficient related to the QCD beta function and quark mass anomalous dimensions.
- Radiative Corrections are One Loop and One Mass Scale Simpler (at each order in α_s).
- Reproduces the physics well almost up to the UV cut-off of the theory, when compared to full QCD Calculations.

So why extend it?

- Certain observables safe from the UV-cutoff, others not.
  - $pp \to H + X$ fairly immune.
  - $pp \to H(p_t > p_{min}) + X$ and $pp \to H + jj + X$ not as much.
- Gauge Invariance: not a goal to achieve but a tool to be used.
  - Means the same operators contribute to several processes.
  - Calculate the hard region once.
  - Match with easy hard regions to calculate.
  - Only "simple" one loop calculations left.
- Allows for analytic results on higher loop corrections to Higgs p_t even for sizeable p_t.

What does p_t spectrum look like?

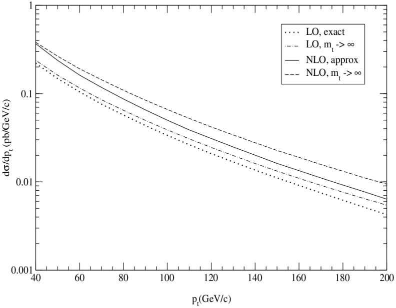

Figure: The p_t spectrum for $m_h = 120 GeV$ at LHC $\sqrt{s} = 14 TeV$

Graphs from Smith, van Neerven, 2005. hep-ph/0501098

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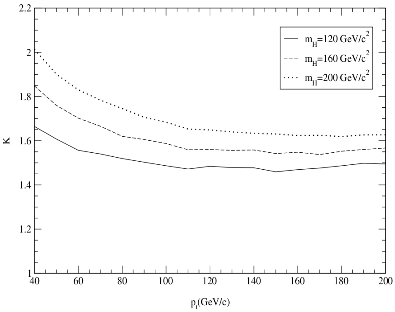

Figure: The K factors for p_t spectrum for $m_h = 120 GeV$ at LHC $\sqrt{s} = 14 TeV$

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So why extend it?

The ratio $R = \frac{\sigma(pp \to H(p_t > p_{min}) + X)}{\sigma(pp \to H + X)}$ is fixed by the standard model for light Higgs. (Arnesen, Rothstein, Zupan, 2009. arXix:0809.1429 [hep-ph])

- R model independent as the same operator governs both processes: matching coefficient cancels.
- But this is true only up to higher order heavy mass corrections. (Or light new physics!)

Onshell new physics breaks down the Higgs Effective Theory.

- Light Colored Octet Scalars coupling to Higgs radically changes cross-section.
- As Octet gets heavier return to Standard Model value set by *HFF* operator.
- Like to understand this transition region in effective theory: New physics or higher order corrections from Standard Model?

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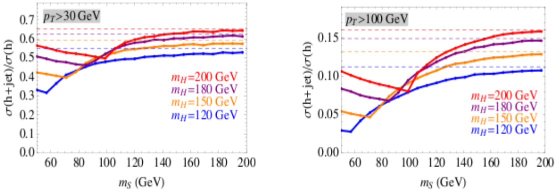


FIG. 4: m_S dependence of R_T for $p_T^H > 30$ GeV (left) and $p_T^H > 100$ GeV (right) for the toy model (solid lines) and for the SM (dashed lines) at LO in α_S. The values of Higgs mass are as indicated. Calculations aided by FeynArts, FormCalc and LoopTools packages [15]. No y_H cut is imposed and κ has been set to unity.

The Extended Lagrangian

To incorporate heavy mass effects at higher loop for the observables $pp \to H(p_t > p_{min}) + X$ and $pp \to H + jj + X$, and to investigate the model dependence of R, extend the effective Lagrangian.

$$
\begin{aligned}
L_{int} &= C_1 \frac{H F^a_{\mu\nu} F^{a\mu\nu}}{4v} + C_2 \frac{H D_\alpha F^a_{\mu\nu} D^\alpha F^{a\mu\nu}}{4vM^2} \\
&+ C_3 \frac{H F^a_{\mu\nu} F^{b\nu}_\sigma F^{c\sigma\mu} f^{abc}}{6vM^2} + C_4 \frac{H F^a_{\mu\nu} D^\nu D_\sigma F^{a\sigma\mu}}{vM^2} \\
&+ C_5 \frac{H D_\mu F^{a\mu}_\nu D_\sigma F^{a\sigma\nu}}{2vM^2} \\
&= C_1 \frac{HFF}{4v} + C_2 \frac{HDFDF}{4vM^2} + C_3 \frac{HFFF}{6vM^2} + C_4 \frac{HFDJ}{vM^2} + C_5 \frac{HJJ}{2vM^2}
\end{aligned}
$$

where J is the QCD current (what one would get upon using equations of motion)

The Extended Lagrangian

- Include operators ($\frac{HF^{a}_{\mu\nu}D^{\nu}D_{\sigma}F^{a\sigma\mu}}{vM^2}$ and $\frac{HD_{\mu}F^{a\mu}_{\nu}D_{\sigma}F^{a\sigma\nu}}{vM^2}$) that can be simplified by equations of motion. This will be helpful in the matching.
- Want C_i to at least two loops in QCD.
- Heavy mass dependence now shows up. At leading order in M, cancels out due to Yukawa coupling.

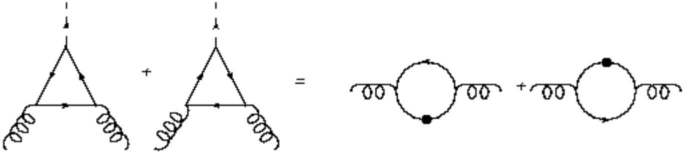
Matching

To calculate the matching coefficients, use the Higgs Low Energy Theorems (LET):

$$\lim_{p_H \to 0} M(X+H) = \frac{g_H}{v} m \frac{\partial}{\partial m} M(X)$$

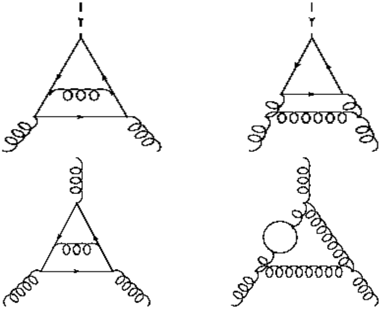
True of unrenormalized amplitudes.

- Only need two processes $gg \to H$ and $gg \to g$ calculated offshell. First was done on-shell to the required order by Dawson and Kauffman in 1993.
- Avoid calculating $gg \to Hg$, naively necessary for $HFFF$ matching.



Matching

- Fix three of the higher dimensional operators from $gg \to H$. Then get $HFFF$ from LET and $gg \to g$. This gives $gg \to Hg$ in the small p_h regime, needed for matching.
- Use method of regions, keeping only hard scale contribution for matching.



Method of Regions

Asymptotically approximate integrals e.g.,
$\int d^d k \frac{1}{(k^2 - m^2)((k+p_1)^2)((k-p_2)^2)}$.

- take m as hard scale, and $(p_1 + p_2)^2 \sim p_1^2 \sim p_2^2 << m^2$
- Two regions of loop integration: hard $k \sim m$ and soft $k \sim p_i$
- In each region, taylor expand integrand (including loop momenta) according to power counting.
- Truncate at desired order, evaluate with *unrestricted* loop momenta, sum regions.
- IR poles from hard region cancel UV poles from soft.

Matching

Checks on calculation.

- Reproduce matching for HFF operator at two loops in both processes calculated.
- Reproduce Dawson and Kauffman's result when taken on-shell.
- Reproduce known anomalous dimensions of the operators, in particular $HFFF$ and $HDFDF$. Spurious infra-red divergences from hard region correspond to UV divergences of effective theory.

Some Numbers

Matching coefficients at scale m_t in $\overline{MS}$.

$$C_1 = \frac{-g^2}{48\pi^2} + \frac{-g^4}{4\pi^4}(\frac{5}{192}C_A - \frac{1}{64}C_F)$$
$$C_2 = \frac{-7g^2}{2880\pi^2} + \frac{-g^4}{4\pi^4}(\frac{29}{34560}C_A + \frac{19}{8640}C_F)$$
$$C_3 = \frac{g^3}{180\pi^2} + \frac{g^5}{6\pi^4}(\frac{49}{9600}C_A + \frac{37}{5760}C_F)$$
$$C_4 = \frac{g^2}{1440\pi^2} + \frac{g^4}{2\pi^4}(\frac{-101}{691200}C_A + \frac{1}{3240}C_F)$$
$$C_5 = \frac{g^2}{80\pi^2} + \frac{g^4}{\pi^4}(\frac{1169}{518400}C_A + \frac{73}{51840}C_F)$$

Calculating In Effective Theory

Naive Expectations:

- Passarino-Veltman reduction unwieldy for higher dimensional operators.
- Ugly operator mixing once renormalized.
- Must calculate for each operator, sum.

Calculating In Effective Theory

Easier than expected with use of unitarity, spinor-helicity methods, equations of motion, and some operator identities.
With equations of motion, re-write basis:

$$\frac{HD^{\alpha}F^{a}_{\alpha\nu}D_{\beta}F^{a\beta\nu}}{vM^2} = \sum_{i=1,j=1}^{n_f} -g^2 \frac{H\bar{q}_i\gamma_{\mu}T^a q_i \bar{q}_j \gamma^{\mu} T^a q_j}{vM^2}$$

$$\frac{HF^{a}_{\alpha\nu}D^{\nu}D^{\beta}F^{a}_{\beta\alpha}}{vM^2} = \sum_{i=1}^{n_f} -ig \frac{HF^{a}_{\alpha\nu}D^{\nu}\bar{q}_i\gamma^{\alpha}T^a q_i}{vM^2}$$

Calculating in Effective Theory

Operator identities give

$$\frac{HD_\alpha F^a_{\mu\nu} D^\alpha F^{a\mu\nu}}{vM^2} = -\frac{H\partial_\alpha \partial^\alpha (F^a_{\mu\nu} F^{a\mu\nu})}{2vM^2} + 4\frac{HF^a_{\mu\nu} D^\nu D_\sigma F^{a\sigma\mu}}{vM^2} - 2\frac{HF^a_{\mu\nu} F^{b\nu}_\sigma F^{c\sigma\mu} f^{abc}}{vM^2}$$

- On-shell the $\partial^2 \to m_h^2$. Effectively, basis reduced by one operator!

Calculating in Effective Theory: Unitarity

- At one loop, one can reduce any amplitude to a known set of master scalar integrals, $A = \sum_i c_i I_i$ where c_i rational coefficients of invariants.
- Cutting both sides, turns left into product of tree amplitudes integrated over two particle d-dimensional Lorentz invariant phase space.
- Reduce the cut left side to make it match cut right side.
- Read off what c_i must be.
- In massless theories in d-dimensions, all terms have an associated cut, thus all terms determined.

Calculating In Effective Theory: Unitarity

- Need d-dimensions as on-shell recursion generically fails for non-renormalizable theories.
- Recycle tree amplitudes.
- Some subtlies, as tree amplitudes in 4-dimensions do not extrapolate to d-dimensions

Calculating In Effective Theory: Spinor-Helicity

Spinor-Helicity methods.

- Write all null vectors as Weyl spinors: e.g.,
  $p^\mu \rightarrow \frac{1}{2}\overline{u}_-(p)\gamma^\mu u_-(p)$
- Vector products take as spinor products:
  $2p.q \rightarrow \overline{u}_-(p).u_+(q)\overline{u}_+(q).u_-(p)$
- Short hand: $\overline{u}_-(p_i).u_+(p_j) =< ij >$ and $\overline{u}_+(p_i).u_-(p_j) = [ij]$
- Take polarization vectors at specific helicities,
  $\epsilon^\mu_\pm(p,q) = \pm\frac{< q\mp|\gamma^\mu|p\mp >}{< q\mp|p\pm >}$

Calculating In Effective Theory: Spinor-Helicity

Why do this?

- Gauge amplitudes compact when using spinors. Vectors and generic polarizations have more information than needed.
- Spinor integration to perform tensor reduction. (Britto et al.) 4-d integration over null vectors $\int d^4 L \delta^+(L^2)$ becomes an integration over left and right handed spinors, and a normalization: $\int_0^\infty tdt \int < LdL > [LdL]$

Calculating In Effective Theory: Spinor-Helicity

Spinor Integration: $\int d^4L\delta^+(L^2) \to \int_0^\infty tdt \int < LdL > [LdL]$

- In cut diagrams (4-d), loop measure is $\int d^4L\delta^+(L^2)\delta^+((L-K)^2)$.
- Second δ takes care of normalization integral.
- Left with integration over spinors, but this is essentially complex integration $\int dzd\bar{z}$, just extract residues!
- Generalizes to d-dimensions as long as external states are 4-d (t'Hooft-Veltman and related schemes).

Calculating In Effective Theory

Example: Partial amplitudes for $gg \rightarrow Hg$ through $HFFF$ operator.

$$M_{tree}(1^+,2^+,3^+) = \frac{[12][23][31]}{vM^2}$$

$$M_{tree}(1^+,2^+,3^-) = 0$$

$$M_{1-loop}(1^+,2^+,3^+) = M_{tree}(1^+,2^+,3^+) r_\Gamma N_c g^2 \left(\left(\frac{1}{\epsilon^2} - \frac{2}{\epsilon}\right)\left(\left(\frac{-m_h^2}{-S_{12}}\right)^\epsilon + \left(\frac{-m_h^2}{-S_{23}}\right)^\epsilon + \left(\frac{-m_h^2}{-S_{13}}\right)^\epsilon\right)\right) - 12)$$

$$M_{1-loop}(1^+,2^+,3^-) = 0$$

With $S_{ij} = 2p_i.p_j$ and $r_\Gamma = \dfrac{\Gamma(1+\epsilon)\Gamma^2(1-\epsilon)}{\Gamma(1-2\epsilon)}$.

Summary

- Still room for work in Higgs production calculations. Extending the Effective Lagrangian a good way to organize the calculations.
- Nice analytic results attainable.

Still to do...

- Calculate cross-sections for Higgs p_t spectrum with subleading corrections.
- Examine the ratio R in effective theory.
- Calculate one-loop corrections for $pp \to H + jj$

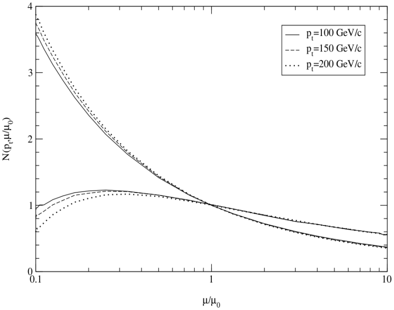


Figure: Renorm. scale variation at p_t=100,150,200 GeV for $m_h = 120 GeV$ at LHC $\sqrt{s} = 14 TeV$ Upper Curves L.O. exact, lower NLO approx.

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